Journal of Engineering Physics and Thermophysics, Vol. 74, No. 6, 2001

THE OPTIMUM THICKNESS OF A COOLED COATED WALL EXPOSED TO LOCAL PULSE-PERIODIC HEATING

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UDC 536.2

In the present work, the optimum thickness of a plane wall with a heat-shielding coating that ensures the minimum steady-state temperature of the most heated point of the wall is determined. On the unprotected side the wall is cooled by a medium whose temperature and heat-transfer coefficient are constant, whereas on the side of the coating the wall is exposed to a heat flux with an intensity of the Gaussian type in the pulse-periodic regime.

In numerous investigations of the mathematical theory of heat conduction [1–4], of special interest are the problems of predicting the thermal state of structures and developing their efficient thermal protection [5–7], which is associated with optimization and estimation of the effective values of thermophysical and geometric parameters for multilayer regions [8]. As the simplest example of a multilayer region, one can consider a plane isotropic wall with a heat-shielding coating which is a layer of a heat-insulating material deposited on a thermally insulated surface and intended for decreasing conductive, convective, and radiative heat exchanges on the surface. Heat insulations are classified using various principles [8, 9], while their comparative analysis is carried out by means of the "effective thermophysical characteristics of a heat-shielding layer" [8, 9], which makes it possible to apply methods of mathematical simulation to calculating and optimizing the coating.

The correctness of selection of the parameters of a heat-shielding coating is substantially determined by the space-time structure of the heat flux which affects the coating. In theoretical investigations, considerable attention is given to heat fluxes with intensities of the Gaussian-type in both steady-state [11–13] and unsteady [14, 15] regimes of exposure. In particular, the use of the heat flux with Gaussian intensity allowed one to solve the problem of determining the "optimum thickness of a cooled wall exposed to local heating" [12].

In the present work, we consider a plane isotropic wall with a heat-shielding coating, whose unprotected surface is cooled by a medium with a constant temperature T_c and a heat-transfer coefficient α, while on the side of the coating the wall is exposed to a heat flux with an intensity of the Gaussian-type in a pulse-periodic regime. The main aim of the investigations performed is to determine sufficient conditions of existence of the optimum thickness of the plane isotropic wall with a coating, which can ensure the minimum steady-state temperature of the most heated point of the wall, with subsequent determination of this optimum thickness.

In conformity with the aim set and the principle of optimality formulated, we consider that the initial temperature in the system wall–coating is equal to the temperature T_c of the cooling medium and use the following mathematical model:

Scientific-Research Institute of Special Mechanical Engineering at N. É. Bauman Moscow State Technical University, Moscow, Russia. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 74, No. 6, pp. 82–87, November–December, 2001. Original article submitted April 6, 2001.

$$\frac{\partial\theta_1}{\partial \mathrm{Fo}} = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\,\frac{\partial\theta_1}{\partial\rho}\right) + \frac{\partial^2\theta_1}{\partial x^2}\,, \quad \mathrm{Fo} > 0\,, \quad \rho \ge 0\,, \quad 0 < x < l\,;$$

$$\frac{\partial\theta_2}{\partial \mathrm{Fo}} = \chi^2\left(\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\,\frac{\partial\theta_2}{\partial\rho}\right) + \frac{\partial^2\theta_2}{\partial x^2}\right), \quad \mathrm{Fo} > 0\,, \quad \rho \ge 0\,, \quad -\varepsilon < x < 0\,;$$

$$\theta_m\,(x, \rho, \mathrm{Fo})\,\big|_{\mathrm{Fo}=0} = 0\,; \quad m = 1, 2\,;$$

$$\left.\frac{\partial\theta_1}{\partial x}\right|_{x=l} = -\,\mathrm{Bi}\;\theta\,\big|_{x=l}\,; \tag{1}$$

$$\theta_1\,\big|_{x=+0} = \theta_2\,\big|_{x=-0}\,, \quad \left.\frac{\partial\theta_1}{\partial x}\right|_{x=+0} = \Lambda\,\left.\frac{\partial\theta_2}{\partial x}\right|_{x=-0}\,;$$

$$\left.\frac{\partial\theta_2}{\partial x}\right|_{x=-\varepsilon} = -\,K\exp\,(-\,k^2\rho^2)\times$$

$$\times\sum_{n=0}^{\infty}\left\{\eta\,[\mathrm{Fo} - n\,(\mathrm{Fo}^* + \Delta\mathrm{Fo}^*)] - \eta\,[\mathrm{Fo} - \mathrm{Fo}^* - n\,(\mathrm{Fo}^* + \Delta\mathrm{Fo}^*)]\right\},$$

where for any fixed values of $\mathrm{Fo} > 0$ and $x \in (-\varepsilon;\, l)$ the function $\theta_m(x, \rho, \mathrm{Fo})$, $m \in \{1, 2\}$, as the function ρ, is the inverse transform of the Hankel integral transform of zero order [1];

$$\theta_m = \frac{T_m - T_c}{T_c}\,, \quad m \in \{1, 2\}\,; \quad x = \frac{z}{z^*}\,; \quad \rho = \frac{r}{z^*}\,; \quad \mathrm{Fo} = \frac{a_1 t}{(z^*)^2}\,; \quad k = k^* z^*\,;$$

$$\mathrm{Bi} = \frac{\alpha z^*}{\lambda_1}\,; \quad \chi^2 = \frac{a_2}{a_1}\,; \quad l = \frac{l_1}{z^*}\,; \quad \varepsilon = \frac{l_2}{z^*}\,; \quad \Lambda = \frac{\lambda_2}{\lambda_1}\,; \quad K = \frac{q_0 z^*}{\lambda_2 T_c}\,.$$

By applying successively to Eq. (1) first the operator H_0 of the Hankel direct integral transform of zero order in ρ [1]

$$U_m\,(x, p, \mathrm{Fo}) = H_0\,[\theta_m\,(x, \rho, \mathrm{Fo})] \equiv \int_0^{\infty} \theta_m(x, \rho, \mathrm{Fo})\,J_0\,(p\rho)\,\rho d\rho\,, \tag{2}$$

and then the operator L of the Laplace direct integral transform in Fo [1]

$$V_m\,(x, p, s) = L\,[U_m\,(x, p, \mathrm{Fo})] \equiv \int_0^{\infty} \exp\,(-\,s\,\mathrm{Fo})\;U_m\,(x, p, \mathrm{Fo})\;d\,\mathrm{Fo} \tag{3}$$

with the use of their properties and the known equality [16]

$$\int_0^{\infty} \exp\,(-\,k^2\rho^2)\;J_0\,(p\rho)\;\rho d\rho = \frac{1}{2k^2}\exp\left(-\,\frac{p^2}{4k^2}\right).$$

we come to the following problem for a system of ordinary differential equations relative to the transforms $V_m(x, p, s)$, $m \in \{1, 2\}$:

$$\frac{\partial^2 V_1}{\partial x^2} = (s + p^2)\, V_1 , \quad 0 < x < l ; \tag{4}$$

$$\frac{\partial^2 V_2}{\partial x^2} = \left(\frac{s}{\chi^2} + p^2\right) V_2 , \quad -\varepsilon < x < 0 ; \tag{5}$$

$$\left.\frac{\partial V_1}{\partial x}\right|_{x=l} = -\,\mathrm{Bi}\, V\big|_{x=l} ; \tag{6}$$

$$\left.\frac{\partial V_2}{\partial x}\right|_{x=-\varepsilon} = -\frac{K}{2k^2} \exp\left(-\frac{p^2}{4k^2}\right) \frac{1 - \exp(-s\,\mathrm{Fo}^*)}{s\left\{1 - \exp[-s\,(\mathrm{Fo}^* + \Delta\mathrm{Fo}^*)]\right\}} ; \tag{7}$$

$$V_1\big|_{x=+0} = V_2\big|_{x=-0} , \quad \left.\frac{\partial V_1}{\partial x}\right|_{x=+0} = \Lambda \left.\frac{\partial V_2}{\partial x}\right|_{x=-0} . \tag{8}$$

Solutions of Eqs. (4) and (5) can be represented in the form [1]

$$V_1(x, p, s) = c_{11}(p, s) \exp(-x\sqrt{s + p^2}\,) + c_{12}(p, s) \exp(x\sqrt{s + p^2}\,) , \quad 0 < x < l ; \tag{9}$$

$$V_2(x, p, s) = c_{21}(p, s) \exp\left(-x\sqrt{\frac{s}{\chi^2} + p^2}\right) + c_{22}(p, s) \exp\left(x\sqrt{\frac{s}{\chi^2} + p^2}\right), \quad -\varepsilon < x < 0 , \tag{10}$$

and must satisfy both the boundary conditions (6) and (7) and the conjugation conditions (8), thus leading to a system of linear algebraic equations for finding the functionals $c_{ij}(p, s)$, $i, j \in \{1, 2\}$:

$$\sqrt{s + p^2}\left\{c_{11}(p, s) \exp(-l\sqrt{s + p^2}\,) - c_{12}(p, s) \exp(l\sqrt{s + p^2}\,)\right\} =$$

$$= \mathrm{Bi}\left\{c_{11}(p, s) \exp(-l\sqrt{s + p^2}\,) - c_{12}(p, s) \exp(l\sqrt{s + p^2}\,)\right\} ;$$

$$\sqrt{\frac{s}{\chi^2} + p^2}\left\{c_{21}(p, s) \exp\left(\varepsilon\sqrt{\frac{s}{\chi^2} + p}\right) - c_{22}(p, s) \exp\left(-\varepsilon\sqrt{\frac{s}{\chi^2} + p}\right)\right\} =$$

$$= \frac{K}{2k^2} \exp\left(-\frac{p^2}{4k^2}\right) \frac{1 - \exp(-s\,\mathrm{Fo}^*)}{s\left\{1 - \exp[-s\,(\mathrm{Fo}^* + \Delta\mathrm{Fo}^*)]\right\}} ;$$

$$c_{11}(p, s) + c_{12}(p, s) = c_{21}(p, s) + c_{22}(p, s) ;$$

$$\sqrt{s + p^2}\left\{c_{11}(p, s) - c_{12}(p, s)\right\} = \Lambda \sqrt{\frac{s}{\chi^2} + p^2}\left\{c_{21}(p, s) - c_{22}(p, s)\right\} . \tag{11}$$

Solution of the problem of determination of the temperature field for the coated wall considered in the integral transforms of Laplace (3) and Hankel (2) used is completed by definition of the functionals $c_{ij}(p, s)$, $i, j \in \{1, 2\}$, that satisfy system (11), which follows immediately from equalities (9) and (10). But to achieve the aim set, it is sufficient to know only the steady-state temperature of the most heated point of the wall. Physically, it is quite obvious that in the notation of the mathematical model (1) this temperature is represented by $\theta_1(0, 0, \infty)$.

According to Eqs. (9) and (11), we have

$$V_1(0, p, s) = c_{11}(p, s) + c_{12}(p, s) =$$

$$= \left\{ 1 + \frac{\sqrt{s+p^2} - \mathrm{Bi}}{\sqrt{s+p^2} + \mathrm{Bi}} \exp\left(-2l\sqrt{s+p^2}\right) \right\} c_{11}(p, s) ;$$

$$c_{11} = \frac{\Lambda K}{sk^2} \exp\left(-\frac{p^2}{4k^2} - \varepsilon\sqrt{\frac{s}{\chi^2} + p^2}\right) \varphi^{-1}(p, s) \frac{1 - \exp(-s\,\mathrm{Fo}^*)}{1 - \exp[-s(\mathrm{Fo}^* + \Delta\mathrm{Fo}^*)]} ;$$

$$\varphi(p, s) = \Lambda \sqrt{\frac{s}{\chi^2} + p^2} \left[1 - \exp\left(-2\varepsilon\sqrt{\frac{s}{\chi^2} + p^2}\right)\right] \times$$

$$\times \left(1 + \frac{\sqrt{s+p^2} - \mathrm{Bi}}{\sqrt{s+p^2} + \mathrm{Bi}} \exp\left(-2l\sqrt{s+p^2}\right)\right) + \sqrt{s+p^2} \left[1 + \exp\left(-2\varepsilon\sqrt{\frac{s}{\chi^2} + p^2}\right)\right] \times$$

$$\times \left(1 - \frac{\sqrt{s+p^2} - \mathrm{Bi}}{\sqrt{s+p^2} + \mathrm{Bi}} \exp\left(-2l\sqrt{s+p^2}\right)\right),$$

where $V_1(0, p, s)$ is the Laplace integral transform (3) for the function $U_1(0, p, \mathrm{Fo})$, i.e., $U_1(0, p, \mathrm{Fo}) = L^{-1}[V_1(0, p, s)]$. Using the well-known limiting theorem of operational calculus [1], we find

$$U_1(0, p, \infty) = \lim_{s \to 0} sV_1(0, p, s) = \frac{\Lambda K \mathrm{Fo}^*}{k^2(\mathrm{Fo}^* + \Delta\mathrm{Fo}^*)\,p} \exp\left(-\frac{p^2}{4k^2} - p\varepsilon\right) \psi^{-1}(p) \times$$

$$\times \left\{ 1 + \frac{p - \mathrm{Bi}}{p + \mathrm{Bi}} \exp(-2lp) \right\} ;$$

$$\psi(p) = \Lambda\,[1 - \exp(-2\varepsilon p)] \left(1 + \frac{p - \mathrm{Bi}}{p + \mathrm{Bi}} \exp(-2lp)\right) + [1 + \exp(-2\varepsilon p)] \left(1 - \frac{p - \mathrm{Bi}}{p + \mathrm{Bi}} \exp(-2lp)\right). \tag{12}$$

Thus, the unknown steady-state temperature of the most heated point of the wall considered is determined as

$$\theta_1(0, 0, \infty) = H_0^{-1}[U_1(0, p, \infty)]\big|_{\rho=0} = \int_0^\infty U_1(0, p, \infty)\, p\,dp =$$

$$= \frac{\Lambda K \mathrm{Fo}^*}{k^2(\mathrm{Fo}^* + \Delta\mathrm{Fo}^*)} \int_0^\infty \exp\left(-\frac{p^2}{4k^2} - p\varepsilon\right) \psi^{-1}(p) \left\{ 1 + \frac{p - \mathrm{Bi}}{p + \mathrm{Bi}} \exp(-2lp) \right\} dp , \tag{13}$$

where the functional $\psi(p)$ is defined by equality (12). Here, Eqs. (13) and (12) yield the dependence of this temperature on the parameters of the initial mathematical model (1), including the thickness l of the wall considered and the thickness ε of its coating. Therefore, we set

$$\theta\,(l, \varepsilon) = \theta_1\,(0, 0, \infty)\,. \tag{14}$$

As the physical substantiation of the existence of the optimum (in terms of the above-formulated principle of optimality, which in this case coincides with the optimization criterion) thickness of the wall, we can cite the reasoning from [12]: "With increase in the wall thickness l, two oppositely acting factors come into force: the spread of heat from the central point and the increase in the thermal resistance to the transfer of heat from the heated surface to a cooled surface. With a certain wall thickness the joint action of these factors can lead to a minimum value of the temperature for the central point on the surface heated."

Having used equalities (12)–(14) and performing an immediate check, we ascertain that

$$\left.\frac{\partial\theta\,(l, \varepsilon)}{\partial\varepsilon}\right|_{\varepsilon>0} < 0\,, \quad \exists \lim_{\varepsilon\to\infty} \frac{\partial\theta\,(l, \varepsilon)}{\partial\varepsilon} = -\,0\,,$$

i.e., with any fixed thickness of the wall the increase in the thickness of the coating leads to a monotonic decrease in the sought temperature $\theta(l, \varepsilon)$. Thus, it is worthwhile to seek the condition of existence of the optimum thickness of the wall with a fixed thickness of the coating. Therefore, the scale unit z^* of the spatial variables x and ρ in the mathematical model (1) is assumed to be equal to the fixed thickness of the coating l_{coat}, i.e., it is taken that $\varepsilon = 1$. In this case, l is defined as the wall thickness-to-coating thickness ratio.

Differentiating the integral on the right-hand side of equality (13) with respect to the parameter l for $\varepsilon = 1$ and taking into account equalities (12) and (14), we find the condition needed for the existence of the local extremum:

$$\frac{k^2\,(\text{Fo}^* + \Delta\text{Fo}^*)}{4\Lambda K\text{Fo}^*} \left.\frac{\partial\theta\,(l, \varepsilon)}{\partial l}\right|_{\varepsilon=1} =$$

$$= \int_0^\infty \exp\left(-\frac{p^2}{4k^2} - p\right)[1 + \exp\,(-\,2p)]\,\frac{\exp\,(-\,2lp)\,(\text{Bi}^2 - p^2)}{(p + \text{Bi})^2\,\psi^2\,(p)}\,pdp = 0\,.$$

But since the physical considerations imply that

$$\exists \lim_{l\to\infty} \left.\frac{\partial\theta\,(l, \varepsilon)}{\partial l}\right|_{\varepsilon=1} = +\,0\,,$$

for the minimum of the function $\theta(l, 1)$ to exist the following condition must be satisfied:

$$\exists \lim_{l\to+0} \left.\frac{\partial\theta\,(l, \varepsilon)}{\partial l}\right|_{\varepsilon=1} < 0$$

or, which is the same,

$$J = \int_0^\infty \exp\left(-\frac{p^2}{4k^2} - p\right)\frac{[1 + \exp\,(-\,2p)]\,(\text{Bi}^2 - p^2)\,pdp}{\{\Lambda\,[1 - \exp\,(-\,2p)]\,p + \text{Bi}\,[1 + \exp\,(-\,2p)]\}^2} < 0\,. \tag{15}$$

Next, we suppose that

$$J_1 = \int_0^{\mathrm{Bi}} \exp\left(-\frac{p^2}{4k^2} - p\right) \frac{[1 + \exp(-2p)]\,(\mathrm{Bi}^2 - p^2)\, p dp}{\left\{\Lambda\,[1 - \exp(-2p)]\, p + \mathrm{Bi}\,[1 + \exp(-2p)]\right\}^2}\,, \tag{16}$$

$$J_2 = \int_{\mathrm{Bi}}^{\infty} \exp\left(-\frac{p^2}{4k^2} - p\right) \frac{[1 + \exp(-2p)]\,(p^2 - \mathrm{Bi}^2)\, p dp}{\left\{\Lambda\,[1 - \exp(-2p)]\, p + \mathrm{Bi}\,[1 + \exp(-2p)]\right\}^2}\,. \tag{17}$$

In this case, according to equalities (16) and (17), we have the inequalities $J_1 > 0$ and $J_2 > 0$, while condition (15) can be represented as follows: $J_1 < J_2$.

Thus, if $J_1^* > J_1$ is the upper limit for J_1 and $J_2^* < J_2$ is the lower limit for J_2, then the inequality

$$J_1^* < J_2^* \tag{18}$$

prescribes a sufficient condition for the existence of the optimum thickness of the coated wall under consideration. Here

$$J_1 < \int_0^{\mathrm{Bi}} \exp\left(-\frac{p^2}{4k^2} - p\right) \frac{[1 + \exp(-2p)]\,(\mathrm{Bi}^2 - p^2)\, p dp}{\left\{\mathrm{Bi}\,[1 + \exp(-2p)]\right\}^2} <$$

$$< \frac{1}{\mathrm{Bi}^2} \int_0^{\mathrm{Bi}} \exp\left(-\frac{p^2}{4k^2} - p\right) (\mathrm{Bi}^2 - p^2)\, p dp < \frac{1}{\mathrm{Bi}^2} \int_0^{\mathrm{Bi}} \exp(-p)\,(\mathrm{Bi}^2 - p^2)\, p dp\,,$$

$$J_2 > \int_{\mathrm{Bi}}^{\infty} \exp\left(-\frac{p^2}{4k^2} - p\right) \frac{(p^2 - \mathrm{Bi}^2)\, p dp}{[1 + \exp(-2p)]\,(\Lambda p + \mathrm{Bi})^2} >$$

$$> \frac{1}{2} \int_{\mathrm{Bi}}^{\infty} \exp\left(-\frac{p^2}{4k^2} - p\right) \frac{(p + \mathrm{Bi})\, p}{(\Lambda p + \mathrm{Bi})^2}\,(p - \mathrm{Bi})\, dp > \frac{1}{(\Lambda + 1)^2} \int_{\mathrm{Bi}}^{\infty} \exp\left(-\frac{p^2}{4k^2} - p\right) (p - \mathrm{Bi})\, dp\,,$$

and if we assume that

$$J_1^* = \frac{1}{\mathrm{Bi}^2} \int_0^{\mathrm{Bi}} \exp(-p)\,(\mathrm{Bi}^2 - p^2)\, p dp = (\mathrm{Bi})^{-2} \left\{(\mathrm{Bi}^2 - 6) + 2\,(\mathrm{Bi}^2 + 3\,\mathrm{Bi} + 3)\exp(-\mathrm{Bi})\right\}; \tag{19}$$

$$J_2^* = \frac{1}{(\Lambda + 1)^2} \int_{\mathrm{Bi}}^{\infty} \exp\left(-\frac{p^2}{4k^2} - p\right) (p - \mathrm{Bi})\, dp =$$

$$= \frac{2k^2 \exp(k^2)}{(\Lambda + 1)^2} \left\{\exp\left[-\left(\frac{\mathrm{Bi}}{2k} + k\right)^2\right] - \sqrt{\pi}\left(\frac{\mathrm{Bi}}{2k} + k\right) \mathrm{erfc}\left(\frac{\mathrm{Bi}}{2k} + k\right)\right\}, \tag{20}$$

then the sufficient condition (18)–(20) is fully defined. It should be noted that for large values of $x = k + \mathrm{Bi}/(2k)$ in calculating the limit J_2^*, determined by equality (20), it is worthwhile to use the asymptotic representation of the complementary Gauss error function [16]

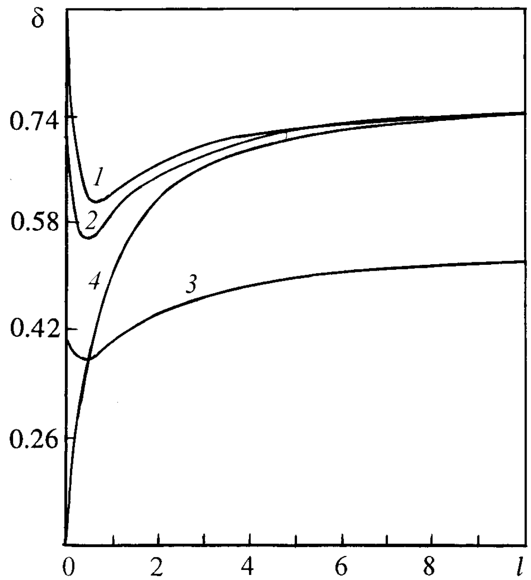

Fig. 1. Dependence of the dimensionless parameter δ on the relative thickness l of the wall at different values of Biot (Bi) criterion and the concentration coefficient k for a pulse-periodic heat flux with intensity of the Gaussian type: 1) Bi = 1, k = 100; 2) 1.3 and 100; 3) 1 and 1; 4) 5 and 100.

$$\mathrm{erfc}\,(x) \approx \frac{\exp\,(-x^2)}{x\sqrt{\pi}}\left\{1 + \sum_{k=1}^{\infty} (-1)^k \frac{(2k-1)\,!!}{(2x^2)^k}\right\}.$$

In this case, with an accuracy to the second term in the asymptotic representation erfc (x) we have

$$J_2^* \approx \frac{\exp\left[-\mathrm{Bi} - \dfrac{\mathrm{Bi}^2}{4k^2}\right]}{(\Lambda + 1)^2\left[1 + \dfrac{\mathrm{Bi}}{2k^2}\right]^2}. \qquad (21)$$

To illustrate the theoretical results obtained, we assume that the material of the wall is titanium (λ_1 = 15 W/(m·K)) [13]. Then, taking λ_2 = 0.45 W/(m·K) for the heat-shielding coating [17], we obtain Λ = 0.03. We note that for the selected value of the parameter Λ, the sufficient conditions (18)–(21) are satisfied, for example, when k = 100 and Bi = 1 and 1.3, and they are not satisfied when k = 100 and Bi = 5. Therefore, each indicated value of the vector of the parameters (Λ, k, Bi) must correspond to the optimum thickness of the wall with a fixed thickness of the heat-shielding coating. This fact is confirmed by the results of computational experiments (Fig. 1, dependences 1, 2, and 4, respectively). For the convenience of representation of graphic information, along the horizontal axis we plotted here the relative thickness l of the wall, while along the vertical axis, we plotted $\delta = [k^2(\mathrm{Fo}^* + \Delta\mathrm{Fo}^*)/(K\Lambda\mathrm{Fo}^*)]\theta(0, 0, \infty)$, i.e., the quantity that is proportional to the steady-state temperature of the most heated point of the wall under consideration.

But since conditions (18)–(21) are sufficient, theoretically there can exist cases not satisfying these conditions which, however, can ensure the existence of the optimum thickness of the wall (see Fig. 1, dependence 3).

Thus, in conformity with the results of the computational experiments that are partially given in Fig. 1, we can assert that in the case of the existence of the optimum thickness of the coated wall considered, i.e., in the case of the existence of opt $\{l\}$, the increase in the value of Bi for a fixed k is accompanied by a decrease in the value of opt $\{l\}$ up to zero (see Fig. 1, dependences 1, 2, and 4). The decrease in the value of k corresponding to the increase in the dispersion of the affecting flux is accompanied by a sharp increase in the value of opt $\{l\}$ for a fixed Bi (see Fig. 1, dependences 1 and 3). These arguments are "physically transparent" and completely correspond to the fundamental propositions of the mathematical theory of heat conduction [1, 2].

NOTATION

r and z, spatial variables; t, time; T, temperature; $x = z/z^*$ and $\rho = r/z^*$, dimensionless spatial variables; $\mathrm{Fo} = a_1 t/(z^*)^2$, Fourier number; $\mathrm{Bi} = \alpha z^*/\lambda_1$, Biot criterion; $\eta(\mathrm{Fo})$, Heaviside function; z^*, selected scale unit; l_1, wall thickness; l_2, coating thickness; q_0, intensity (density) of the heat flux; k^*, λ, and a, coefficients of concentration, thermal conductivity, and thermal diffusivity; α, heat-transfer coefficient. Subscripts: 1, wall; 2, coating; c, cooling medium.

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